1. Evaluate $\log_2 \left( 1 + \frac{1}{2} \sum_{k=1}^{6} \binom{7}{k} \right)$.

\[ (C) \ 6 \]

2. For any number $x$, we define $x^+$ and $x^-$ such that $x^+ + x^- = |x|$ and $x^+ - x^- = x$. Find the sum $\sum_{j=1}^{100} (2j)^+ - (2j + 1)^-)$.

\[ (A) \ 10100 \]

3. For all real numbers $a, b$ and some fixed real number $n$, the operation $\ast$ is defined as $a \ast b = a^2 - b * n$. Given that $11 \ast 11 = 5$, what is the value of $20 \ast 3$?

\[ (B) \ 396 \]

4. Suppose that $x$ and $y$ are real numbers that satisfy $|x| + \{y\} = 3.2$, and $\{x\} + |y| = 1.7$, where $|x|$ represents the greatest integer less than or equal to $x$, and $\{x\} = x - |x|$. Find the difference $x - y$.

\[ (D) \ 2.5 \]

5. Isosceles $\triangle ABC$ is such that $|AB| = |AC| = \sqrt{\sin \theta}$, $|BC| = \sin \theta$, where $\theta$ is the measure of $\angle A$. Find the area of this triangle.

\[ (A) \ \frac{8}{25} \]

6. Define $f : \mathbb{R} \rightarrow \mathbb{R}$, with $f(x) = \frac{5}{1 + 31e^x}$. How many different integer values can $f(x)$ take?

\[ (C) \ 4 \]

7. In Ms. Brakebill’s calculus 2 class there are 20 students. Furthermore, we know there are 8 female students, 7 freshmen, and 8 math majors. We also know that 3 students are female and freshmen, 4 students are freshmen and math major, 3 students are female and math major, and 1 student who is a freshman, math-major female. How many students in Ms. Brakebill’s class are neither female nor freshmen nor math major?

\[ (E) \ 6 \]

8. Evaluate the sum $\sum_{n=1}^{\infty} [\arctan(n + 1) - \arctan(n - 1)]$.

\[ (D) \ \frac{3\pi}{4} \]

9. How many values $-\infty < x < +\infty$ satisfies $x[|x|] = 8$, where $|x|$ denotes the largest integer less than or equal to $x$?

\[ (B) \ 1 \]
10. How many functions $f : \mathbb{R} \to \mathbb{R}$ satisfy, for all real $x, y$, $f(x) + f(y) = xf(y)$?

$$\text{(B) 1}$$

11. A friend has a stack of ten cards numbered 1 through 10. He shuffles the cards and you draw two cards. What is the probability that the cards are consecutive numbers?

$$\text{(B) \frac{1}{5}}$$

12. Points $A, B, C, D$ are on the plane so that $\angle ABC + \angle ADC = 180^\circ$. Perpendicular bisectors to $AB$ and $AD$ intersect at point $P$. If $|PC| = 17$, find $|PB|$.

$$\text{(C) 17}$$

13. If $x = \frac{1002}{1003}$, for what positive integer $n$ is the following equation satisfied?

$$x + 2x + \ldots + nx = x^2 + 2x^2 + \ldots + (n + 1)x^2.$$ 

$$\text{(B) 2004}$$

14. How many zeros are at the end of $\binom{100}{51}$?

$$\text{(B) 2}$$

15. How many points of positive integer coordinates are on the ellipse $2x^2 + 13y^2 = 213$?

$$\text{(B) 1}$$

16. At how many points do $y = \sin x$ and $y = \frac{1}{5\pi} x$ intersect?

$$\text{(D) 11}$$

17. A 3-digit palindrome is a 3-digit number (not starting with 0) which reads the same backwards and forwards (for example, 171). Find the sum of all even 3-digit palindromes.

$$\text{(B) 22000}$$

18. How many pairs $(a, b)$ of positive integer numbers are solution of the equation $a^2 - b^2 = 105$?

$$\text{(E) 4}$$

19. Given a square with sidelength 1, determine the radius of the smaller circle $C_2$ given $C_1$’s radius is twice that of $C_2$’s, and these circles are tangent to each other and the square.

$$\text{(B) \frac{\sqrt{2}}{3(\sqrt{2} + 1)}}$$

20. A regular octahedron has volume $V \neq 1$. Its surface area can be written as $S = 2^a3^bV^c$, where $a$, $b$, and $c$ are rational numbers. Find $a + b + c$.

$$\text{(A) \frac{5}{2}}$$