Ciphering Round Junior Varsity League

High School Math Competition 2007

Georgia Institute of Technology

February 24th, 2007
Problem #1

Problem

How many pairs of integers \((m, n)\) are there such that \(mn = m + n\)?
Problem #1

Problem

How many pairs of integers \((m, n)\) are there such that \(mn = m + n\)?

Answer

1
Problem #2

If a certain number is reduced by 7 and the remainder is multiplied by 7, the result is the same as if the number is reduced by 11 and the remainder is multiplied by 11. What is the number?
Problem #2

Problem

If a certain number is reduced by 7 and the remainder is multiplied by 7, the result is the same as if the number is reduced by 11 and the remainder is multiplied by 11. What is the number?

Answer

18 = 7 + 11
Problem #3

If $x$ and $y$ are distinct positive real numbers, which ratio is greater,

$$\frac{x^2 + y^2}{x + y} \quad \text{or} \quad \frac{x^2 - y^2}{x - y}?$$
Problem #3

Problem
If \( x \) and \( y \) are distinct positive real numbers, which ratio is greater,

\[
\frac{x^2 + y^2}{x + y} \quad \text{or} \quad \frac{x^2 - y^2}{x - y}.
\]

Answer

\[
\frac{x^2 + y^2}{x + y} \leq \frac{x^2 - y^2}{x - y}.
\]
Problem #4

A father budgeted $24 to distribute equally among his children for spending money at the beach. When two young cousins joined the party and shared in equal distribution, each child received $1 less than originally planned. How many kids were in the party?
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Problem

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Answer

8 kids
Problem #5

Problem

How many positive integer solutions \((a, b, c)\) are there to the equation

\[2004^a + 2005^b = 2006^c?\]
Problem #5

**Problem**

How many positive integer solutions \((a, b, c)\) are there to the equation

\[2004^a + 2005^b = 2006^c?\]

**Answer**

None (one side is odd, the other is even)
A new edition of a book is published every seven years. When the seventh edition was issued, the sum of the publication years was 13524. When was the book first published?
Problem #6

Problem

A new edition of a book is published every seven years. When the seventh edition was issued, the sum of the publication years was 13524. When was the book first published?

Answer

1926
Problem #7

Problem

There is a real number \( x > 1 \) such that, if I add it to its reciprocal, and then I add what I get to its reciprocal, I end up with the sum equal to 27. Find \( \lfloor x \rfloor \), the integer part of \( x \).
Problem #7

Problem

There is a real number $x > 1$ such that, if I add it to its reciprocal, and then I add what I get to its reciprocal, I end up with the sum equal to 27. Find $\lfloor x \rfloor$, the integer part of $x$.

Answer

26
Problem #8

Problem

Find a function \( f(x) \neq x \) such that for every \( x \geq 0 \),

\[
f \left( \frac{x}{1+x} \right) = \frac{f(x)}{1 + f(x)}.
\]
Problem #8

Problem

Find a function $f(x) \neq x$ such that for every $x \geq 0$,

$$f \left( \frac{x}{1+x} \right) = \frac{f(x)}{1 + f(x)}.$$  

Answer

One is $f(x) = \frac{x}{1 + x}$.
Problem #9

Problem

If $x$, $y$ and $z$ are non-zero real numbers such that $x - y = xy = \frac{x}{y} = z$, find all possible values of $z$. 
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If $x$, $y$ and $z$ are non-zero real numbers such that $x - y = xy = \frac{x}{y} = z$, find all possible values of $z$.

Answer

$z = \frac{1}{2}$
Problem #10

Problem

Some friends are trying to divide a pile of stones. They try to split the stones into three equal groups, but end up with 1 stone left over. Surprisingly, the same thing happens when the try splitting the stones into equal groups of 4, 5, 6, and 7; there is always 1 stone left over! What is the fewest number of stones that the friends could have (beyond just 1 stone of course!)?
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Answer

421
THE END