Ciphering Round Varsity League

High School Math Competition 2008

Georgia Institute of Technology

February 23\textsuperscript{rd}, 2008
Problem #1

Problem

How many different squares of integer side length can be formed grouping together the cells (little squares in the grid) if each cell has side length 1.

Answer

\[ 175 = 10 \cdot 6 + 9 \cdot 5 + 8 \cdot 4 + 7 \cdot 3 + 6 \cdot 2 + 5 \cdot 1 \]
Problem #1

Problem

How many different squares of integer side length can be formed grouping together the cells (little squares in the grid) if each cell has side length 1.

Answer

\[ 175 = 10 \cdot 6 + 9 \cdot 5 + 8 \cdot 4 + 7 \cdot 3 + 6 \cdot 2 + 5 \cdot 1 \]
Problem #2

Problem

How many permutations of the name of the letters in the name

“GEORGE P BURDELL”

are there? (Ignore the spaces!)
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How many permutations of the name of the letters in the name “GEORGE P BURDELL” are there? (Ignore the spaces!)

Answer

\[
\frac{14!}{3!(2!)^3}
\]
Problem #3

Problem

Find all the points \((r, \theta)\), with \(0 \leq \theta \leq \pi\), where the curves \(r^2 = \sin \theta\) and \(r = 2 - \sin \theta\) intersect.
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Find all the points \((r, \theta)\), with \(0 \leq \theta \leq \pi\), where the curves \(r^2 = \sin \theta\) and \(r = 2 - \sin \theta\) intersect.

Answer

\[
\left(1, \frac{\pi}{2}\right)
\]
Problem #4

Find the simplified value of

\[
\frac{6508^2 - 2492^2}{4501^2 - 4499^2}.
\]
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Problem

Find the simplified value of

\[
\frac{6508^2 - 2492^2}{4501^2 - 4499^2}.
\]

Answer

\[
\frac{6508^2 - 2492^2}{4501^2 - 4499^2} = \frac{(6508 - 2492)(6508 + 2492)}{(4501 - 4499)(4501 + 4499)} = 2008
\]
Γ is a circle with radius \( r \). It is inscribed in a 60° sector of a circle with radius \( R \). Find \( \frac{R}{r} \).
Problem #5

**Problem**

Γ is a circle with radius $r$. It is inscribed in a $60^\circ$ sector of a circle with radius $R$. Find $R/r$.

**Answer**

$$R/r = 3$$
Problem #6

Given that $a, b, c$ and $d$ are positive integers and that

$$a + \frac{1}{b + \frac{1}{c + \frac{1}{d}}} = \frac{181}{42},$$

find $a + b + c + d$. 

Answer

$4 + 3 + 4 + 3 = 14$
Problem #6

Problem

Given that $a, b, c$ and $d$ are positive integers and that

\[ a + \frac{1}{b + \frac{1}{c + \frac{1}{d}}} = \frac{181}{42}, \]

find $a + b + c + d$.

Answer

\[ 4 + 3 + 4 + 3 = 14 \]
Problem #7

If $a, b, 10, c, d$ are in geometric progression, find $abcd$. 
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Problem
If $a, b, 10, c, d$ are in geometric progression, find $abcd$.

Answer
$$abcd = 10^4 = 10,000$$
Find the center of mass of the collection of points \{A, B, C, D, E\} if A, C and E each have mass \(M\) and B and D each have mass \(2M\).
Problem #8

Problem

Find the center of mass of the collection of points \{A, B, C, D, E\} if A, C and E each have mass \(M\) and B and D each have mass \(2M\).

\[ A = (-1, 0) \]
\[ C = (0, 1) \]
\[ B = (3, 0) \]
\[ D = (2, 1) \]
\[ E = (1, 2) \]

Answer

\[ \left( \frac{10}{7}, \frac{5}{7} \right) \]
Problem #9

Problem

Of the 8-letter strings in which each letter is either A or B, how many contain the pattern \(ABBA\) exactly once?
Problem #9

Of the 8-letter strings in which each letter is either $A$ or $B$, how many contain the pattern $ABBA$ exactly once?

Answer

70
Problem #10

Given the following binary numbers:

\[ \begin{align*}
    a_1 &= 101 \\
    a_2 &= 1011 \\
    a_3 &= 10111 \\
    a_4 &= 101111 \\
    a_5 &= 1011111 \\
    a_6 &= 10111111 \\
\end{align*} \]

Evaluate \( \sum_{k=1}^{6} a_k \), and express your answer in base 10.
Problem #10

Problem

Given the following binary numbers:

\[
\begin{align*}
    a_1 &= 101 \\
    a_2 &= 1011 \\
    a_3 &= 10111 \\
    a_4 &= 101111 \\
    a_5 &= 1011111 \\
    a_6 &= 10111111 \\
\end{align*}
\]

Evaluate \( \sum_{k=1}^{6} a_k \), and express your answer in base 10.

Answer

372