TIE BREAKERS

0.1. **Problem #1.** Sum the infinite series:

\[ \sum_{i=1}^{\infty} \frac{1}{(3i-2)(3i+1)}. \]

**Solution.** The answer is 1/3. We obtain a telescoping series where \( S_n = \frac{1}{3}\left[(1 - \frac{1}{4}) + \left(\frac{1}{4} - \frac{1}{7}\right) + \cdots + \left(\frac{1}{3n-2} - \frac{1}{3n+1}\right)\right]. \) As \( n \) approaches \( \infty \) the limit of this sum is 1/3.

0.2. **Problem #2.** How many integers between 2009 to 9002 are perfect squares?

**Solution.** The answer is 50. Notice that \( 45^2 = 2025 \) and \( 95^2 = 9025 \). So 45 is included and 95 is excluded.
0.3. **Problem #3.** Write $x$, $y$, and $z$ in order from smallest to largest if $x = 2^{100}$, $y = 3^{75}$, and $z = 5^{50}$.

Solution. $x < z < y$.

0.4. **Problem #4.** Let $x_1, x_2, \ldots, x_7$ be seven distinct real numbers such that $\max_{i<j} |x_i - x_j| = 1$. Find the minimum value $C$ such that for any such collection

$$\sum_{1 \leq i < j \leq 7} |x_i - x_j| < C.$$

Solution. 12 (In general, if there are $2k + 1$ points, $C = k^2 + k$, it is an limit that is never reached).
0.5. **Problem #5.** How many triples \((p,q,r)\) are there such that \(p\) is a member of the set \(\{1, 2, 3, 4, 5, 6\}\), \(q\) is a member of the set \(\{7, 8, 9, 10, 11, 12\}\), \(r\) is a member of the set \(\{13, 14, 15, 16, 17, 18\}\), and \(p + q + r = 32\)?

**Solution.** \(15 = \binom{6}{2}\)